Control of Discrete Systems

ENSICA

Yves Briere
ISAE

General Introduction

Overview

Prerequisites :

Automatique ENSICA 1A et 2A
Traitement numérique du signal ENSICA 1A
(Control and signal processing, basics)

Tools :
Matlab / Simulink

References :

- « Commande des systèmes », *I. D. Landau, Edition Lavoisier 2002.*

Planning

22 slots of 1h15

Overview

Discrete signals and systems

Sampling continuous systems

Identification of discrete systems

Closed loop systems

Control methods

Control by computer

I. Introduction

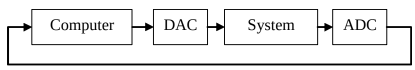
II. Discrete signals and systems

Signal processing / Control

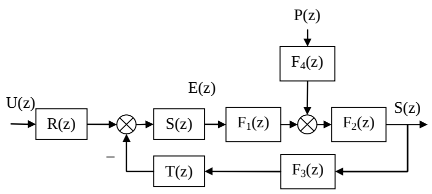
→ Signal processing gives tools to describe and filter signals
→ Control theory use these tools to deal with closed loop systems
→ More generally, control theory deal with :
  → discrete state system analysis and control (Petri nets, etc...)
  → Complex systems, UML, etc...

II. Discrete signals and systems

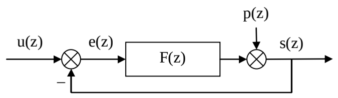
What we deal with in this course



The same problem as seen by the control system engineer :



Or, simply :



II. Discrete signals and systems

II. Discrete signals and systems

Reminder : the z-transform

Discrete signal : list of real numbers (samples)

$$s(k) = \{s_0, s_1, s_2, \ldots\}$$

Z-transform : function of the complex z variable

$$s(z) = Z(s(k)) = \sum_{k=0}^{\infty} s(k) \cdot z^{-k}$$

Existence of s(z) : generally no problem (convergence radius : s(z) exist for a given radius |z| > R)

9

II. Discrete signals and systems

Reminder : properties of the z-transform

1	$s(z) = Z(s(k)) = \sum_{k=0}^{\infty} s(k) \cdot z^{-k}$	
2	$Z(\alpha \cdot s_1(k) + \beta \cdot s_2(k)) = \alpha \cdot Z(s_1(k)) + \beta \cdot Z(s_2(k))$	Linearity
3	$Z(k \cdot s(k)) = -z \cdot \frac{ds(z)}{dz}$	
4	$Z(c^k \cdot s(k)) = s\left(\frac{z}{c}\right)$	
5	$Z(s(k-1)) = s(k=-1) + z^{-1} \cdot s(z)$	Delay theorem
6	$Z(s(k+1)) = z \cdot s(z) - z \cdot s(k=0)$	
7	$s(k=0) = \lim_{z \to \infty}(s(z))$	
8	$s(k \to \infty) = \lim_{z \to 1}\left(\frac{z-1}{z} \cdot s(z)\right)$	Final value theorem

10

II. Discrete signals and systems

Reminder : basic signals

s(k)	S(z)
$\delta(k)$: unit impulse	1
u(k) : unit step	$\frac{z}{z-1}$
k.u(k)	$\frac{z}{(z-1)^2}$
$c^k \cdot u(k)$	$\frac{z}{z-c}$
$\sin(\omega \cdot k) \cdot u(k)$	$\frac{z \cdot \sin(\omega)}{z^2 - 2 \cdot z \cdot \sin(\omega) + 1}$

11

II. Discrete signals and systems

Reminder : from « z » to « k »

First approach : use the z-transform equations (we don't give here the inverse z-transform equation, it is too ugly...)

Second approach : use tricks
→ recurrence inversion
→ Polynoms division
→ Singular value decomposition

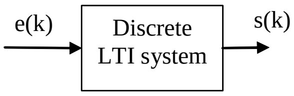
Third approach : computer (Matlab)

12

II. Discrete signals and systems

Reminder : discrete transfer function

We deal with Linear Time Invariant (LTI) systems



The sequence of output and input samples are consequently simply related by:

$$s(k) + a_1 \cdot s(k-1) + a_2 \cdot s(k-2) + \ldots + a_n \cdot s(k-n) =$$
$$b_0 \cdot e(k) + b_1 \cdot e(k-1) + b_2 \cdot e(k-2) + \ldots + b_m \cdot e(k-m)$$

The delay theorem gives:

$$\frac{s(z)}{e(z)} = F(z) = \frac{b_0 + b_1 \cdot z^{-1} + b_2 \cdot z^{-2} + \ldots + b_m \cdot z^{-m}}{1 + a_1 \cdot z^{-1} + a_2 \cdot z^{-2} + \ldots + a_n \cdot z^{-n}}$$

Normalized : $a_0 = 1$

13

II. Discrete signals and systems

Reminder : discrete transfer function

Remarks

→ We prefer to use z^{-1} rather than z. (z^{-1} is a « shift » operator)

→ We often use the q^{-1} notation instead of z^{-1} : this way we don't bother with radius convergence and other fundamental mathematic stuff.

→ Impulse response : e(z) = 1
The transfer function is <u>also</u> the impulse response
(function = signal)

→ Causality : the output depends on past, not future
→ the impulse response is null for $k < 0$
→ Confusion between « causal system » and « causal signal »

14

II. Discrete signals and systems

Reminder : discrete transfer function

Properties

→ Impulse response : the inverse z-transform of the transfer function

→ Step response

$$s(z) = \frac{b_0 + b_1 \cdot z^{-1} + b_2 \cdot z^{-2} + \ldots + b_m \cdot z^{-m}}{1 + a_1 \cdot z^{-1} + a_2 \cdot z^{-2} + \ldots + a_n \cdot z^{-n}} \times \frac{1}{1 - z^{-1}}$$

→ Static gain (Final value theorem applied to the last equation)

$$F_0 = \frac{\sum_{i=0}^{i=m} b_i}{1 + \sum_{i=1}^{i=n} a_i}$$

15

II. Discrete signals and systems

Reminder : discrete transfer function

Properties : stability

→ Any transfer function can be expressed as:

$$F(z) = \frac{e_0 \cdot (1 + e_1 \cdot z^{-1}) \cdot (1 + e_2 \cdot z^{-1}) \cdots (1 + e_m \cdot z^{-1})}{(1 + c_1 \cdot z^{-1}) \cdot (1 + c_2 \cdot z^{-1}) \cdots (1 + c_n \cdot z^{-1})}$$

Coefficients c_i are either real or complex conjugates

For a stable system each c_i coefficient must verify $|c_i| < 1$, in other words each poles must belong to the unit circle.

Properties : singular value decomposition

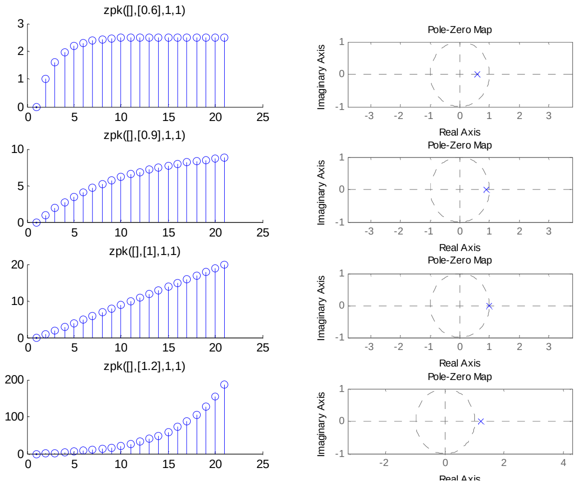
F(z) can be decomposed in a sum of first order and second order systems
→ It is good to know how first and second order behaves

16

II. Discrete signals and systems

Reminder : first order systems

Properties : first order system

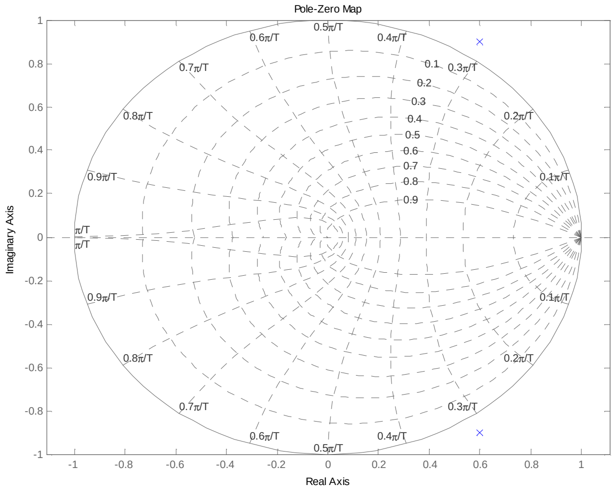


17

II. Discrete signals and systems

Reminder : second order systems

Properties : second order systems

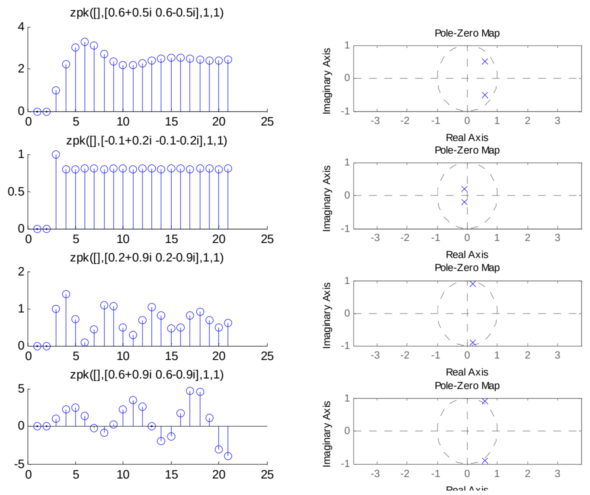


18

II. Discrete signals and systems

Reminder : second order systems

Properties : second order systems



II. Discrete signals and systems

Reminder : frequency response

Reminder : continuous systems

$$\begin{cases} e(t) = \cos(2\cdot\pi\cdot f\cdot t) + j\cdot\sin(2\cdot\pi\cdot f\cdot t) = e^{j\cdot 2\cdot\pi\cdot f\cdot t} \\ f \in [0 \quad \infty] \end{cases}$$

Analogy : discrete system

$$\begin{cases} e(k) = \cos(2\cdot\pi\cdot f\cdot k) + j\cdot\sin(2\cdot\pi\cdot f\cdot k) = e^{j\cdot 2\cdot\pi\cdot f\cdot k} \\ f \in [0 \quad 1] \end{cases}$$

The output signal looks like

$$\begin{cases} s(k) = A(f)\cdot e^{j\cdot(2\cdot\pi\cdot f\cdot k + \Phi(f))} \\ f \in [0 \quad 1] \end{cases}$$

With F of phase Φ (degrees) and module A (decibels):

$$\begin{cases} F(f) = F\left(z = e^{j\cdot 2\cdot\pi\cdot f}\right) \\ f \in [0 \quad 1] \end{cases}$$

II. Discrete signals and systems

Reminder : frequency response

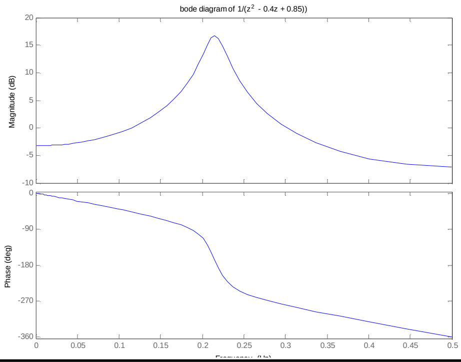
Definition : frequency response

$$\begin{cases} F(f) = F\left(z = e^{j \cdot 2 \cdot \pi \cdot f}\right) \\ f \in [0 \quad 1] \end{cases}$$

f : normalized frequency (between 0 and 1)

F(f) : frequency response

Property : Bode diagram

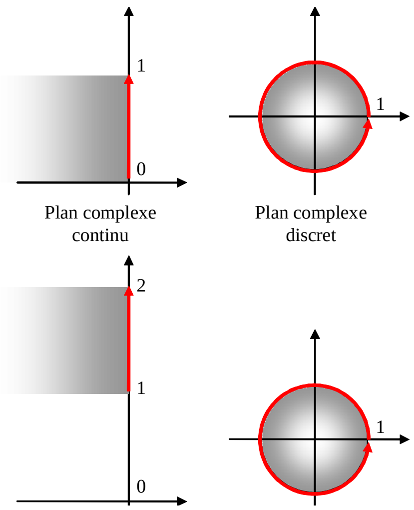


Bode diagram of a second order system. Note the linear scale for frequency.

21

II. Discrete signals and systems

Reminder : Nyquist-Shannon sampling theorem



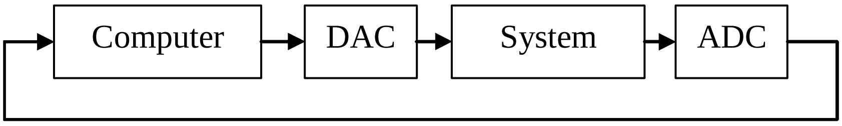
22

End of C1 C2

III. Sampling continuous systems

III. Sampled continuous systems

Discrete systems



A real **system** is generally continuous (and described by differential equations)

An interface provides sampled measurements at a given frequency:

→ Digital to Analog Converter (DAC)
→ smart **sensor**

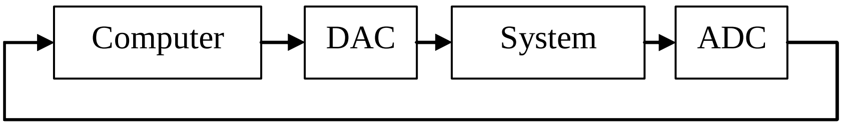
A **computer** provides at a given frequency the input of the system

An interface transforms the computer output into a continuous input for the system

→ Analog to Digital Converter (ADC)
→ smart **actuator**

III. Sampled continuous systems

Hypothesis



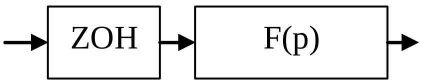
→ Sampling is **regular**
→ Sampling is **synchronous**
→ The computer computes the control according to the curr »ent measurement and a finite set of pas measurements.

III. Sampled continuous systems

Sampling a continuous transfer function

Hypothesis :
→ The continuous transfer function is known
→ The ADC is a ZOH ☺

We must now deal with F+ZOH in a whole

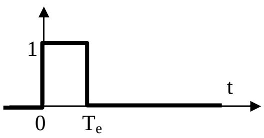


III. Sampled continuous systems

Sampling a continuous transfer function

Zero Holder Hold : the sampled input is blocked during one sampling delay.

The impulse response of a zoh is consequently:



The Laplace transform of the above signal is:

$$ZOH(s) = \frac{1}{s} - \frac{e^{-T_e \cdot s}}{s}$$

III. Sampled continuous systems

Sampling a continuous transfer function

The impulse response of ZOH+F is :

$$s(s)=\left(\frac{1}{s}-\frac{e^{-j\cdot T_e}}{s}\right)\cdot F(s)=\frac{1}{s}\cdot F(s)-e^{-j\cdot T_e}\cdot\frac{1}{s}F(s)$$

Two terms for the impulse response :

$$s(k)=Z\left(\frac{1}{s}\cdot F(s)\right)-z^{-1}\cdot Z\left(\frac{1}{s}F(s)\right)=\left(1-z^{-1}\right)\cdot Z\left(\frac{1}{s}F(s)\right)$$

Usually the z-transfer function of ZOH+F is given by tables:

29

III. Sampled continuous systems

Sampling a continuous transfer function

Tables

$F(s)$	$F_{BOZ}(z)$
1	1
$\frac{1}{s}$	$\frac{T_e\cdot z^{-1}}{1-z^{-1}}$
$\frac{1}{1+T\cdot s}$	$\begin{cases}\frac{b_1\cdot z^{-1}}{1+a_1\cdot z^{-1}}\\ b_1=1-e^{-T_e/T}\quad a_1=-e^{-T_e/T}\end{cases}$
$\frac{e^{-L\cdot s}}{1+T\cdot s}$, $L<T_e$	$\begin{cases}\frac{b_1\cdot z^{-1}+b_2\cdot z^{-2}}{1+a_1\cdot z^{-1}}\quad a_1=-e^{-T_e/T}\\ b_1=1-e^{(L-T_e)/T}\quad b_2=e^{-T_e/T}\left(e^{L/T}-1\right)\end{cases}$

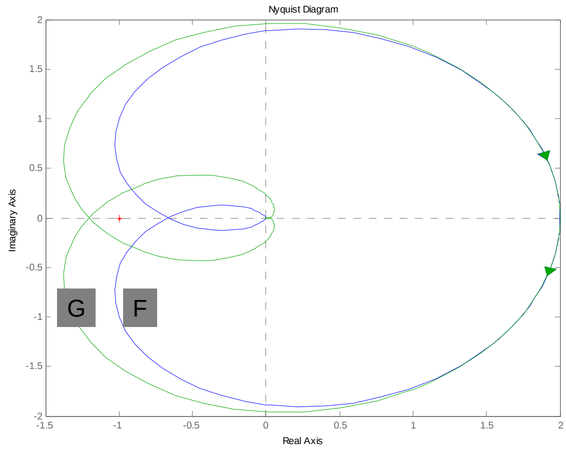
Matlab

```
Matlab
% Definition of a continuous system :
sysc=tf(1,[1 1]);
% z_transfer function after sampling
Te=0.1;
sysd=c2d(sysc,Te,'zoh');
% Notation with z^-1
sysd.variable='z
```

III. Sampled continuous systems

Sampling time delay equivalence

A sampling time of T_e can be approximated as a delay of $T_e/2$: instability in closed loo^p.




```
>> F=2*s/(1+s+s^2)/s/(1+s)
>> G=F;
>> G.ioDelay=0.5
>> nyquist(F,G)
```


III. Sampled continuous systems

Choice of sampling time

The sampling time is chosen according to the **closed loop** expected performance

Sampling frequency : 5 to 25 fois the expected closed loop bandwidth.

Example : third order system

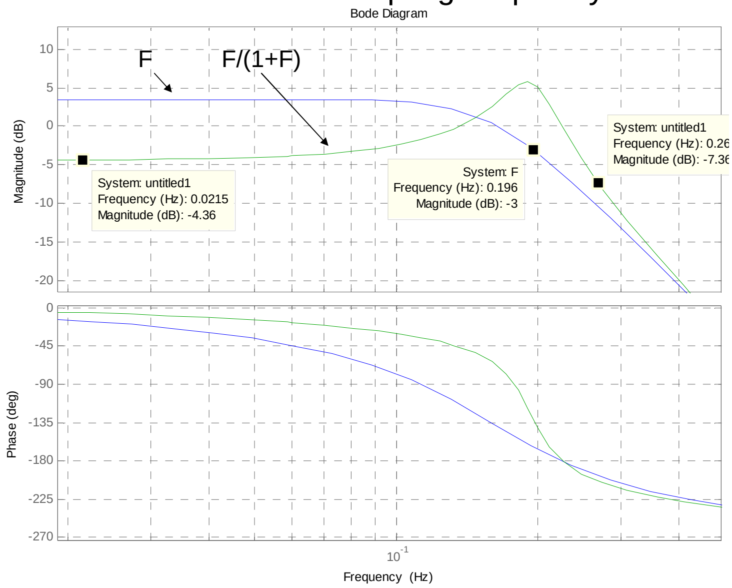
```
>> F=1.5/(1+s+s^2)/(1+s)
```

Closed loop bandwitdh at -3dB of F/(1+F) : 0.3 Hz

Sampling time : 5 Hz

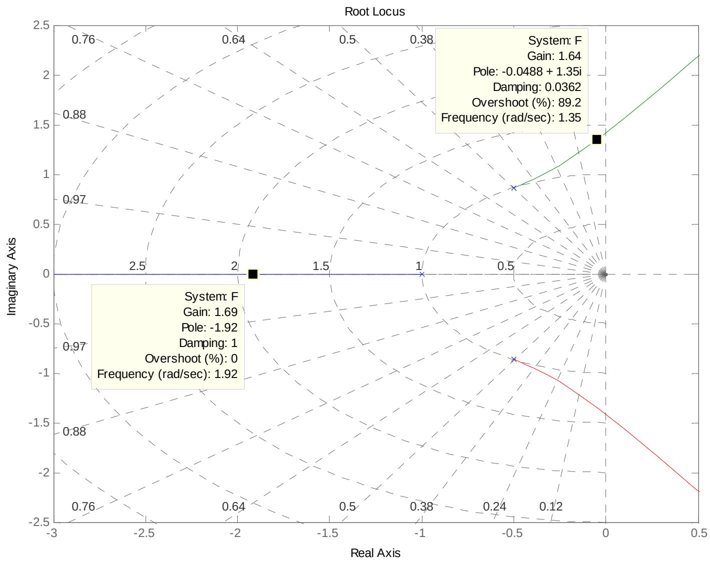
III. Sampled continuous systems

Choice of sampling frequency



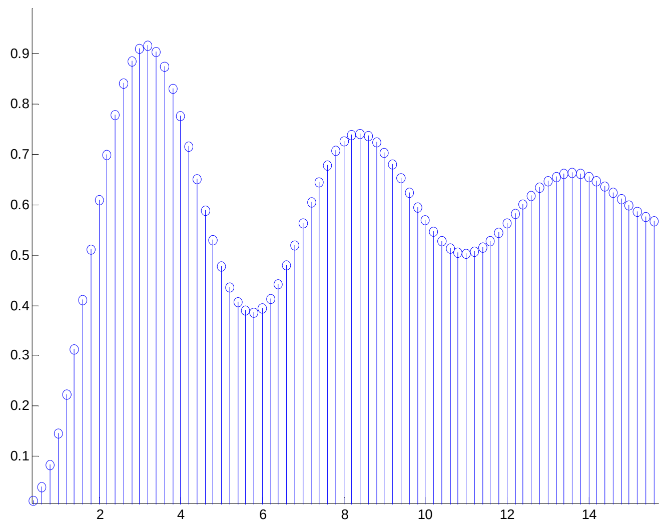
III. Sampled continuous systems

Choice of sampling frequency



III. Sampled continuous systems

Choice of sampling frequency

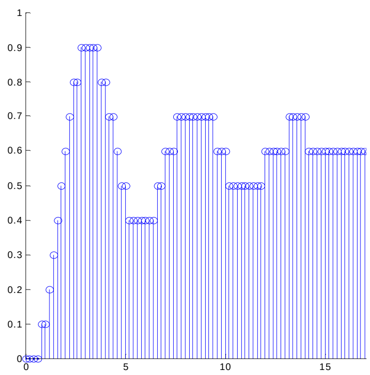


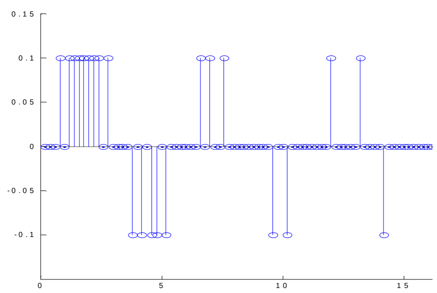
35

III. Sampled continuous systems

Sampling effect : discretization

Sampling is always associated to a discretization of the signal !
Example :
- Signal sampled at 0.2s and discretized at a resolution of 0.1.
- The numerical derivative of the signal





36

III. Sampled continuous systems

First approach : discretise a continuous controller

The sampling effect (ZOH) is neglected.

Example : continuous PD :

$$PID(s) = \frac{e(s)}{\varepsilon(s)} = k_p + k_D \cdot s$$

$$e(t) = k_p \cdot \varepsilon(t) + k_D \cdot \frac{d\varepsilon(t)}{dt}$$

Continuous derivative is replaced by a discrete derivative :

$$e(k) = k_p \cdot \varepsilon(k) + k_D \cdot \frac{\varepsilon(k) - \varepsilon(k-1)}{T_e}$$

Transfer function :

$$e(z) = k_p \cdot \varepsilon(z) + k_D \cdot \frac{\varepsilon(z) - z^{-1}\varepsilon(z)}{T_e} = \left(k_p + k_D \cdot \frac{1 - z^{-1}}{T_e} \right) \cdot \varepsilon(z)$$

S to z equivalence :

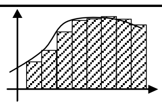
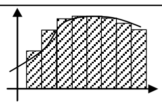
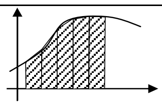
$$s = \frac{1 - z^{-1}}{T_e}$$

37

III. Sampled continuous systems

First approach : discretise a continuous controller

More generally the problem is to approximate a differential equation : cf numerical methods of integration

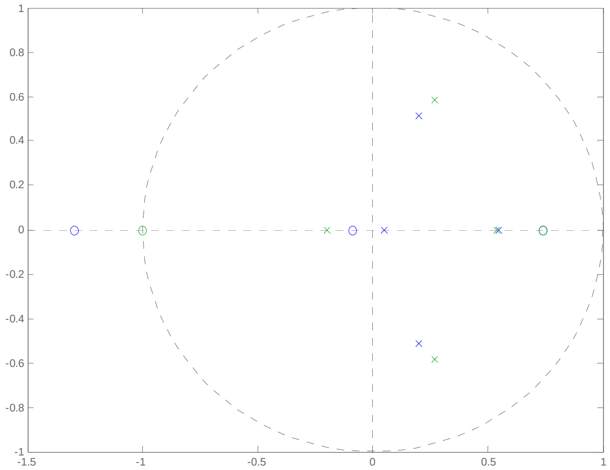
Method	s to z equivalence	
Euler's forward method	$s = \frac{1 - z^{-1}}{z^{-1} \cdot T_e}$	
Euler's backward method	$s = \frac{1 - z^{-1}}{T_e}$	
Tustin method	$s = \frac{2}{T_e} \cdot \frac{1 - z^{-1}}{1 + z^{-1}}$	

38

III. Sampled continuous systems

First approach : discretise a continuous controller

Warning : methods are not equivalent !



Blue : zoh
Green : tustin

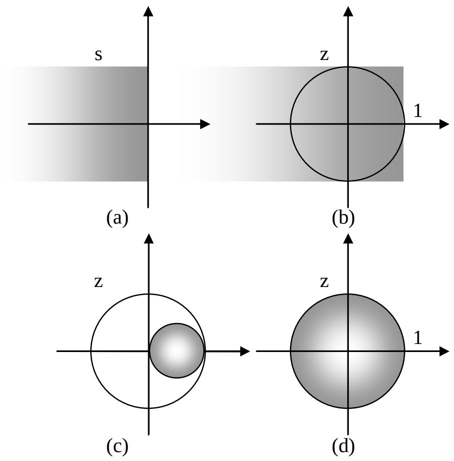
(No problem if sampling time is small)

39

III. Sampled continuous systems

First approach : discretise a continuous controller

Equivalence of contrinuous poles after « s to z » conversion



(a) : continuous
(b) : Euler forward
(c) : Euler backward
(d) : Tustin

40

III. Sampled continuous systems

End C3 C4

IV. Identification of discrete systems

IV. Identification of discrete systems

Introduction

Before the design process of a controller one must have a discrete model of the plant

→ First approach : continuous model and/or identification then discretization (cf last chapter)

→ Second approach : tests on the real system, measurements (sampled), identification algorithm

For a full course on this topic see 3d year course “airplane identification”

43

IV. Identification of discrete systems

Naive approach

(But easy, fast and easy to explain)

→ Standard test (step, impulse)

→ Try to fit as well as possible a model (first order, second order with delay, etc...)

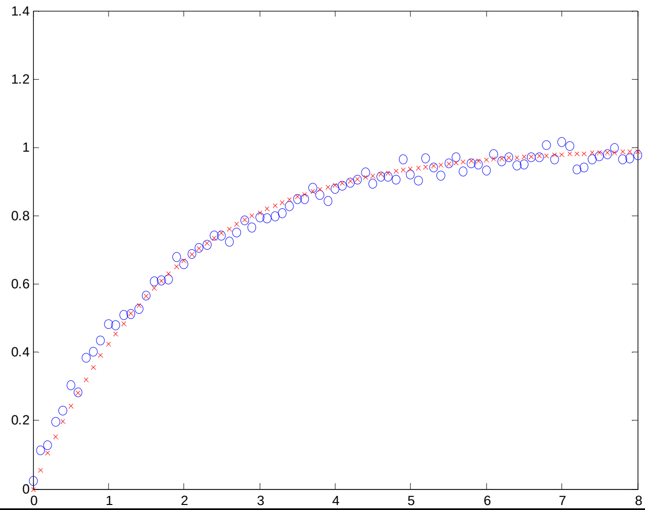
... « fit as well as possible » ...
→ criteria ?...
→ optimisation ?…

44

IV. Identification of discrete systems

Naive approach

Example : real step test (o) and simulated first order step (x)



45

IV. Identification of discrete systems

Naive approach

A little bit of method ?

→ trial and error fitting of the outputs ☹

→ find a criteria ☺ , example : mean square error...

$$J = \frac{1}{N}\sum_{t=1}^{N}\left(y_{reel}(t) - y_{mesuré}(t)\right)^2$$

```
>> t=0:0.1:8;
>> p=tf('p');
>> sysreal=tf(1/(1+2*p)/(1+0.6*p)*(1+0.5*p))
+0.02*randn(length(t),1);
>> yreal=step(sysreal,t)+0.02*randn(length(t),1);
>> ysimul=step(1/(1+2*p),t);
>> plot(t,yreal,'bo',t,ysimul,'rx')


>> eps=yreel-ysimul;
>> J=1/length(eps)*eps'*eps;
```

After some iterations one obtain J minimal with tau=1.81 s

Better model with a delay, two parameters to tune

46

IV. Identification of discrete systems

Naive approach

Drawbacks :

→ Test signals with high amplitude (is the system still linear ?)
→ Reduced precision
→ Perurbation noise not taken into account
→ Perturbation noise reduces the performance of the identification
→ long, not very rigorous...

Advantage :

→ easy to understand...

IV. Identification of discrete systems

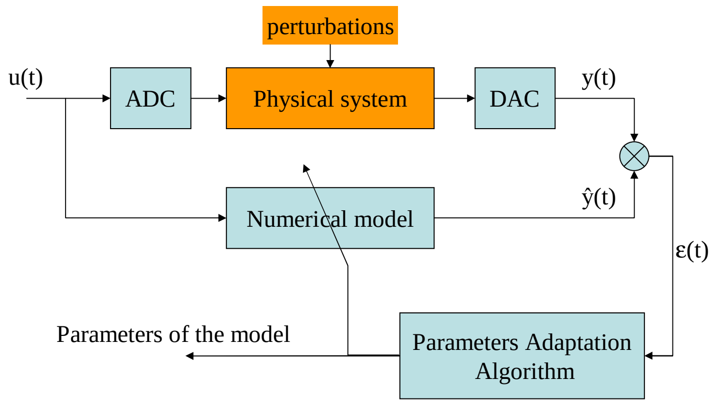
Second method (the good one) : « estimation »

→ Perform a « rich test » (random input signal, rich in frequency)

→ Fit the model that best predict the output.

IV. Identification of discrete systems

Basic principle of parametric identification



49

IV. Identification of discrete systems

Example of an Identification Process

Linear model (ARX) :

$$A \cdot y = B \cdot u + e$$

White noise

$$y(t+1) + a_1 \cdot y(t) + a_2 \cdot y(t-1) + ... + a_{n_a} \cdot y(t-n_a+1) =$$
$$b_1 \cdot u(t) + b_2 \cdot u(t-1) + ... + b_{n_b} \cdot u(t-n_b+1) + e(t)$$

Model used as a **predictor** :

$$y_{predict}(t+1) = -\left(a_1 \cdot y(t) + a_2 \cdot y(t-1) + ... + a_{n_a} \cdot y(t-n_a+1)\right) +$$
$$..............................b_1 \cdot u(t) + b_2 \cdot u(t-1) + ... + b_{n_b} \cdot u(t-n_b+1)$$

→ Try to minimize the difference between prediction and actual measurement.

50

IV. Identification of discrete systems

Example of an Identification Process

Example : orders $n_a = 3$ et $n_b = 2$

measurements		prediction	error
u1	y1		
u2	y2		
u3	y3	yp3	eps3
u4	y4	yp4	eps4
u5	y5	yp5	eps5
u6	y6	yp6	eps6
u7	y7	yp7	eps7
u8	y8	yp8	eps8
		Total	J

$$yp5 = -\left(a_1 \cdot y1 + a_2 \cdot 2 + a_3 \cdot y3\right) +$$
$$\ldots\ldots\ldots\ldots\ldots\ldots b_1 \cdot u1 + b_2 \cdot u2$$

$$eps5 = yp5 - y5$$

$$J = \frac{1}{N}\sum_{t=1}^{N}(eps_t)^2$$

Unknown : coefficients

$\{a_1, a_2, a_3, b_1, b_2\}$

51

IV. Identification of discrete systems

Example of an Identification Process

Recursive Least Square Algorithm

→ Possible since model is linear

→ Possible since quadratic criteria

→ Interesting because of the recursive version

$$y_{predict}(t+1) = -\left(a_1 \cdot y(t) + a_2 \cdot y(t-1) + \ldots + a_{n_a} \cdot y(t-n_a+1)\right) +$$
$$\ldots\ldots\ldots\ldots\ldots\ldots b_1 \cdot u(t) + b_2 \cdot u(t-1) + \ldots + b_{n_b} \cdot u(t-n_b+1)$$

Prediction equation can be re-written :

$$y_{predict} = \theta^T(t) \cdot \phi(t)$$

Where : $\theta(t)^T = [a_1, a_2, \ldots b_1, b_2, \ldots]$

$\varphi(t)^T = [-y(t), -y(t-1), \ldots u(t), u(t-1), \ldots]$

52

IV. Identification of discrete systems

Least Square Algorithm

Estimate θ taking into account the full set of measurements
→ Obtained θ is optimal

$$\hat{\theta} = F(t)\sum_{i=1}^{t}\left(y(i)\cdot\phi(i-1)\right)$$

With :

$$F(t)^{-1} = \sum_{i=1}^{t}\left(\varphi(i-1)\cdot\varphi(i-1)^T\right)$$

53

IV. Identification of discrete systems

Recursive Least Square (rls) Algorithm

Estimate θ recursively for t = 0...N
→θ obtained after recursion is optimal

$$\theta(t) = \theta(t-1) + F(t)\cdot\varphi(t)\cdot\left(y(t) - \theta^T(t-1)\cdot\varphi(t)\right)$$

with :

$$F(t)^{-1} = F(t-1)^{-1} + \varphi(t)\cdot\varphi(t)^T$$

Algorithm can be applied in real time
→ parameters supervision
→ diagnostis
→ Algorithm can be improved with a forget factor : λ=0.95...0.99

$$\theta(t) = \theta(t-1) + F(t)\cdot\varphi(t)\cdot\left(y(t) - \theta^T(t-1)\cdot\varphi(t)\right)$$

with :

$$F(t)^{-1} = \lambda\cdot F(t-1)^{-1} + \varphi(t)\cdot\varphi(t)^T$$

54

IV. Identification of discrete systems

Identification algorithms (general)

Les algorithmes d'identification sont en général des variantes des moindres carrés récursifs
En tous cas ils ont tous, pour les algorithmes récursifs, la forme suivante :

$$\theta(t) = \theta(t-1) + F(t) \cdot \varphi(t) \cdot \left(y(t) - \theta^T(t-1) \cdot \varphi(t)\right)$$

avec

$$F(t)^{-1} = \lambda \cdot F(t-1)^{-1} + \varphi(t) \cdot \varphi(t)^T$$

55

IV. Identification of discrete systems

Identification method

1. Choice of a frequency sampling
   → Not too big
   → Not too small
   → Adapted to the fastet expected dynamic (usually closed loop dynamic)

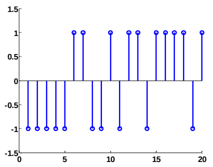
56

IV. Identification of discrete systems

Identification method

2. Excitation signal
   → Le signal doit être riche en fréquence
   → Signal can (and must) be of small amplitude (a few %)

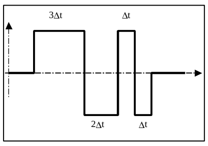
Example 1 : Pseudo Random Binary Signal



Matlab / System Identification Toolboxe

>> u=idinput(500,'prbs')

Example 2 : excitation 3-2-1-1 (airplane identification)



57

IV. Identification of discrete systems

Identification method

3. Test and measurements
   → Choice of a setpoint
   → Test
   → Remove mean of measurements
   → Remove absurd values
4. Model choice
   → Example : ARX, degree of A and B
5. Apply algorithm
   → Example : rls
6. Validate
   → Validation criteria ?

Iterate

58

IV. Identification of discrete systems

Validation criteria

1. The model must **predict** the output
2. The model behavior must be the same than the original for perturbations

The model :

$$y(t+1)+a_1\cdot y(t)+a_2\cdot y(t-1)+...+a_{n_a}\cdot y(t-n_a+1)=$$
$$b_1\cdot u(t)+b_2\cdot u(t-1)+...+b_{n_b}\cdot u(t-n_b+1)+e(t)$$

With the estimated model (a_i, b_i) one can estimate the error !

$$e_{est}(t)=y_{est}(t+1)-y(t+1)$$
$$e_{est}(t)=a_1\cdot y(t)+a_2\cdot y(t-1)+...+a_{n_a}\cdot y(t-n_a+1)$$
$$..............-b_1\cdot u(t)+b_2\cdot u(t-1)+...+b_{n_b}\cdot u(t-n_b+1)$$
$$..............-y(t+1)$$

→ Check that $e_{est}(t)$ is a white noise !

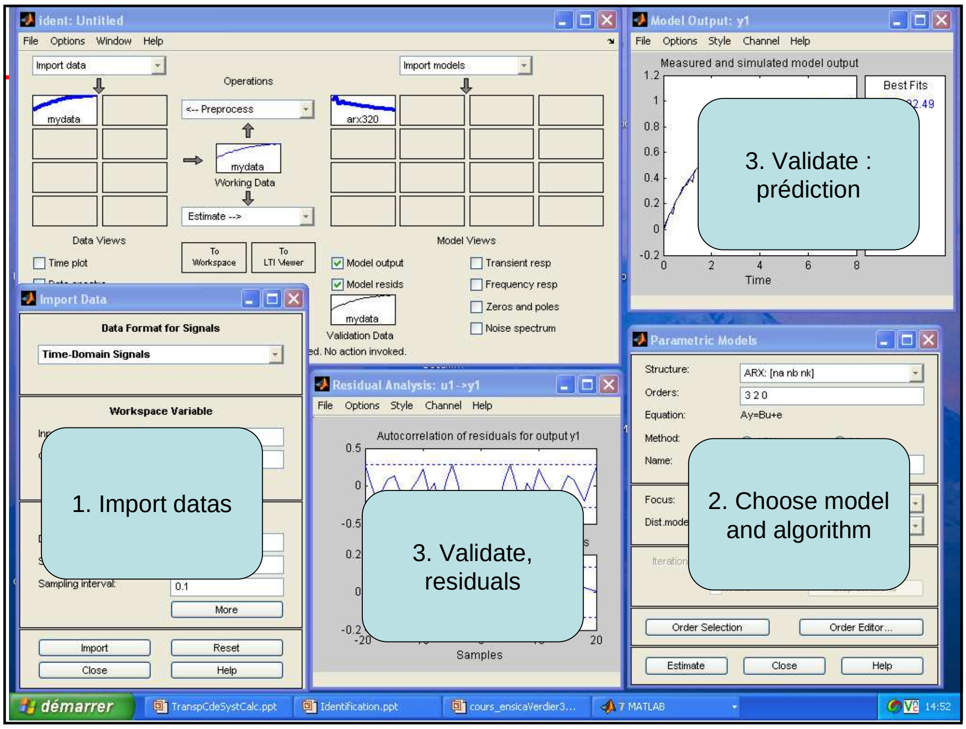
59

IV. Identification of discrete systems

Use dedicated software

Example : Matlab / System Identification Toolboxe...

60



IV. Identification of discrete systems

Use dedicated software

→ Accelerate the identification process

Matlab licence = 3000 € + toolboxes licence...

→ « homemade » toolboxes are used for dedicated applications

IV. Identification of discrete systems

Non parametric identification

From input/output datas, estimate the transfer function :
→ Input / output cross correlation
  → FFT, Hamming window and tutti quanti
→ Gain and phase, Bode diagram

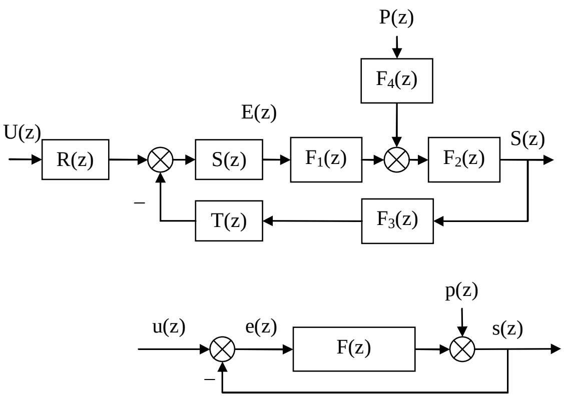
From input / output datas, estimate the **impulse response**
→ Apply RLS algorithm with n_B big and $n_A = 0$
→ Allow for instance to quickly estimate the pure delay

63

V. Closed loop systems

V. Closed loop systems

Reminder



Equivalent !

Open loop transfer function :

$F = S.F_1.F_2.F_{3.T}$

V. Closed loop systems

Closed loop transfer function

Open loop transfer function :

$F = S.F_1.F_2.F_{3.T}$

Closed loop transfer function :

$G = F(1+F)$

$G = B/(A+B)$

V. Closed loop systems

Static gain

As seen before :

$$G_0 = \frac{\sum_{i=0}^{i=m} b_i}{1 + \sum_{i=0}^{i=m} b_i + \sum_{i=1}^{i=n} a_i}$$

Condition to have a unitary closed loop static gain (no static error) :

$$G_0 = \frac{\sum_{i=0}^{i=m} b_i}{1 + \sum_{i=0}^{i=m} b_i + \sum_{i=1}^{i=n} a_i} = 1 \quad \Leftrightarrow \quad \boxed{F(z) = \frac{s(z)}{e(z)} = \frac{1}{1 - z^{-1}} \cdot \frac{B(z)}{A'(z)}}$$

Integrator

V. Closed loop systems

Perturbation rejection

Perturbation-output transfer function

$$S_{yp}(z) = \frac{s(z)}{p(z)} = \frac{A(z^{-1})}{A(z^{-1}) + B(z^{-1})}$$

→ Same condition on A(1) to perfectly reject the perturbations (integrator in the loop)

Generally the rejection perturbation condition is weaker :
→ $|S_{yp}(e^{j2\pi f})| < S_{max}$ for a subinterval of [0 0.5]

V. Closed loop systems

Stability : poles

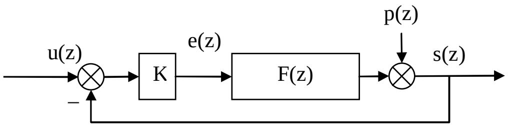
System is stable is the poles belong to the unit circle

→ Compute closed loop transfer function

  → Jury criteria (equivalent to Routh)

  → Compute poles

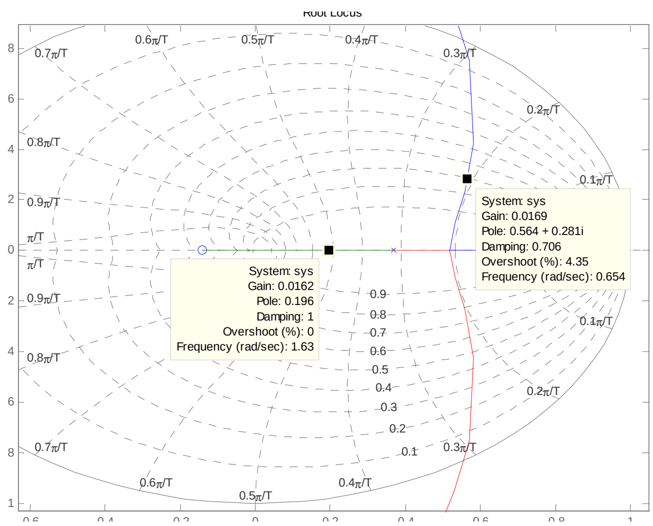
→ Draw the root locus



69

V. Closed loop systems

Example : root locus

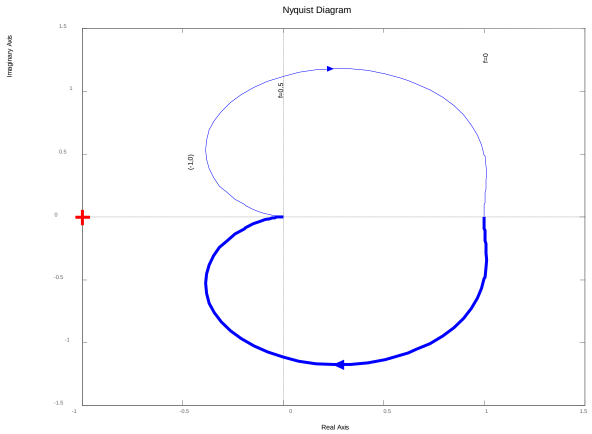


70

V. Closed loop systems

Stability : frequency criteria

Nyquist criteria : as in the continuous case

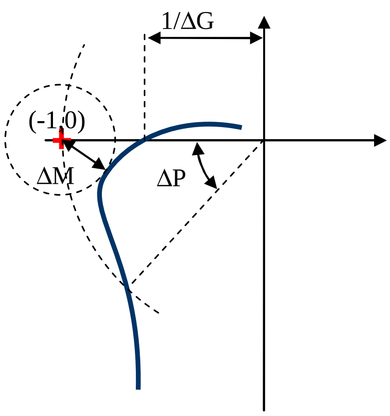


71

V. Closed loop systems

Nyquist criteria

Margins :



- Module margin □ΔM
- Gain module□ΔG
- Phase module□ΔP

72

V. Closed loop systems

end C5 C6

VI. Control of sampled systems

VI. Control of closed loop systems

Hypothesis

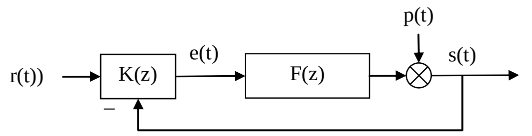
Requisites :

→ Synchronous sampling
→ Regular sampling
→ Sampling time adapted to the desired closed loop performance

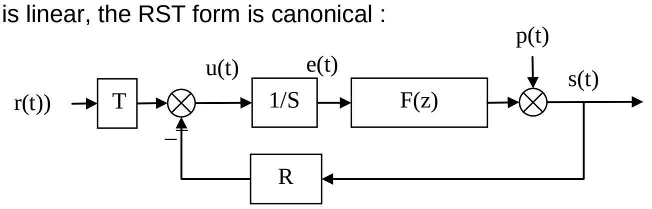
VI. Control of closed loop systems

Introduction

Controller computes e(t) given r(t) and s(t) :



If K is linear, the RST form is canonical :



R, S et T are **polynoms**

VI. Control of closed loop systems

Discretization of a continuous controller

1. Process is modelled as a linear transfer function (Laplace)
   → No need to know a sampled model of the system
2. Design of a continuous controller
3. Discretization of the continuous controller
   → Many discretization methods
   → Problem if T_e too big (stability)
   → Problem if T_e too small (numerical round up)

77

VI. Control of closed loop systems

Design of a discrete controller

1. System is known as a discrete transfer function (z-transform)
   → Discxretizaiton of a continuous model + ZOH
   → Direct identification
2. Design of a discrete controller
   1. PID 1
   2. PID 2
   3. Pole placement
   4. Independant tracking and regulation goals

78

VI. Control of closed loop systems

PID 1

Continuous PID :

$$K_{PID}(s) = K\cdot\left(1+\frac{1}{T_i\cdot s}+\frac{T_d\cdot s}{1+\frac{T_d}{N}\cdot s}\right)$$

Discretization (Euler) :

$$K_{PID}(q)=K\cdot\left(1+\frac{1}{T_i\cdot\frac{1-q^{-1}}{T_e}}+\frac{T_d\cdot\frac{1-q^{-1}}{T_e}}{1+\frac{T_d}{N}\cdot\frac{1-q^{-1}}{T_e}}\right)$$

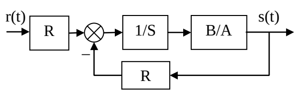
VI. Control of closed loop systems

PID 1

$$K_{PID}(q)=K\cdot\frac{r_0+r_1\cdot q^{-1}+r_2\cdot q^{-2}}{\left(1-q^{-1}\right)\cdot\left(1+s_1^{'}\cdot q^{-1}\right)}$$

After re-arranging :

$$\begin{cases} r_0 = K\cdot\left(1+\frac{T_e}{T_i}+\frac{N\cdot T_d}{T_d+N\cdot T_e}\right) \\ r_1 = -K\cdot\left(1+\frac{T_d}{T_d+N\cdot T_e}+\frac{T_e}{T_i}\cdot\frac{T_d}{T_d+N\cdot T_e}+\frac{N\cdot T_d}{T_d+N\cdot T_e}\right) \\ r_2 = K\cdot\left(\frac{T_d}{T_d+N\cdot T_e}\right) \\ s_1^{'} = \frac{T_d}{T_d+N\cdot T_e} \end{cases}$$



VI. Control of closed loop systems

PID 1

Closed loop transfer function is : :

$$H_{CL} = \frac{B \cdot R}{A \cdot S + B \cdot R} = \frac{B \cdot R}{P}$$

S as an integral teerm (the I of PID) :

$$P = A \cdot S + B \cdot R = A \cdot \left(1 - z^{-1}\right) \cdot S' + B \cdot R$$

Desired dynamics is given by the roots of P :

A,B,P → R S'

Small degrees : by hand
Large degrees : Bezout algorithm

81

VI. Control of closed loop systems

PID 1

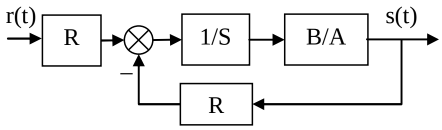
Main drawback : new zeros given by R at the denominator...
Solution : PID2

82

VI. Control of closed loop systems

PID 2

Starting from the already known PID1 :



R is replaced by R(1) :

$$H_{CL} = R(1) \cdot \frac{B}{P}$$

Static gain is one, desired dynamic remains the same
→ Same performances in rejection perturbation
→ Better performance (smaller overshoot) in tracking

83

VI. Control of closed loop systems

Pole placement

Can be seen as a more general PID 2 where degrees of R and S are not constrained.

$$H_{OL} = \frac{q^{-d} \cdot B(q^{-1})}{A(q^{-1})}$$

$$\begin{cases} A(q^{-1}) = 1 + a_1 \cdot q^{-1} + \ldots + a_{n_A} \cdot q^{-n_A} \\ B(q^{-1}) = b_1 \cdot q^{-1} + \ldots + b_{n_B} \cdot q^{-n_B} = q^{-1} \cdot B^*(q^{-1}) \end{cases}$$

Tracking performance

$$H_{CL} = \frac{q^{-d} \cdot B(q^{-1}) \cdot T(q^{-1})}{P(q^{-1})}$$

$$P(q^{-1}) = A(q^{-1}) \cdot S(q^{-1}) + q^{-d} \cdot B(q^{-1}) \cdot R(q^{-1}) = 1 + p_1 \cdot q^{-1} + \ldots + p_{n_P} \cdot q^{-n_P}$$

Regulation performance : $S_{yp} = \frac{A(q^{-1}) \cdot S(q^{-1})}{P(q^{-1})}$

84

VI. Control of closed loop systems

Pole placement

P poles gives the closed loop dynamic :
→ P_D Dominant poles (second order, natural frequency, damping)
→ P_F auxiliary poles, faster

A,B,P → P : compute R and S

Static gain :
→ $S = (1-q^{-1}).S'$

Rejection of harmonic perturbation
→ $S = H_S.S'$ where $|H_S|$ is small at a given frequency

Remove sensibility to an given frequency :
→ $R = H_R.R'$ where $|H_R|$ is small at a given frequency

85

VI. Control of closed loop systems

Pole placement

Perturbation rejection : choice of R and S

Static gain :
→ $S = (1-q^{-1}).S'$

Rejection of harmonic perturbation
→ $S = H_S.S'$ where $|H_S|$ is small at a given frequency

Remove sensibility to an given frequency :
→ $R = H_R.R'$ where $|H_R|$ is small at a given frequency

$P_F.P_D = (1-q^{-1}).H_S.S'.A + H_R.R'.B$

$A.(1-q^{-1}).H_S$ $B.H_R$ $P_F.P_D$ → R' and S'

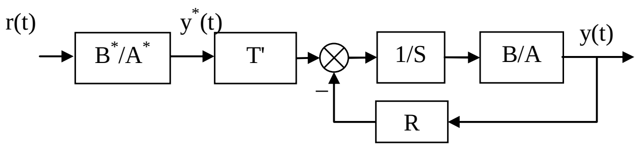
86

VI. Control of closed loop systems

Pole placement

Tracking : choice of T

T is replaced by the normalized factor :



1. T' is chosen such as the transfer y*→y is as « transparent » as possible
2. B*/A* chosen such as r→y* as the desired dynamic

87

VI. Control of closed loop systems

Pole placement

Tracking : choice of T'

$$S_{yy^*}(q^{-1}) = T'(q^{-1})\frac{q^{-d}\cdot B(q^{-1})}{P(q^{-1})}$$

With :

$$T'(q^{-1}) = \frac{P(q^{-1})}{B(1)}$$

One obtain :

$$S_{yy^*}(q^{-1}) = q^{-d}\cdot\frac{B(q^{-1})}{B(1)}$$

88

VI. Control of closed loop systems

Placement de pôle

Tracking : choice of A^* and B^* :

→ chosen according to tracking specifications

89

VI. Control of closed loop systems

Independant tracking and perturbation rejection specifications

P chosen to cancel poles of B^*
→ B must be stable

Tracking : choice of T'

$$S_{yy^*}\left(q^{-1}\right)=T'\left(q^{-1}\right)\frac{q^{-d}\cdot q^{-1}\cdot B^*\left(q^{-1}\right)}{P\left(q^{-1}\right)\cdot B^*\left(q^{-1}\right)}$$

With :

$$T'\left(q^{-1}\right)=P\left(q^{-1}\right)$$

One obtain :

$$S_{yy^*}\left(q^{-1}\right)=q^{-(d+1)}$$

90